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Bayes Solutions of Some Simple Statistical Decision Problems

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Bayes solutions are obtained for some specific simple statistical decision problems which may be stated in the following practical terms. The result of a trial of a weapon system is valued as success (1) or failure (0) and p , the true probability of success is unknown. Prior to testing, it is assumed that any value of p in the interval $(0,1)$ is equally likely. Take as an estimate of p , the fraction, p_e , of a finite number of trials which result in success. At the conclusion of testing, the weapon system is accepted (terminal decision d_1) if $p_e > \theta$, and it is rejected (terminal decision d_2) if $p_e < \theta$, where θ is a number in the interval $(0,1)$ chosen by the experimenter. If $p_e = \theta$, either terminal decision may be made. A wrong decision may be made in two ways: (i) if the weapon system is accepted and the true p lies in an interval $(0,\alpha)$ where $\alpha \leq \theta$ or (ii) if the weapon system is rejected and the true p lies in an interval $(1-\beta,1)$ where $\theta \leq 1-\beta$. In this paper we consider only the case where α and β have the same value which we denote as θ (symmetric case). The cost of wrong decision is defined in terms of a symmetric weight function, $W(p, d_i; \theta)$, $i = 1, 2$; W is simple or linear as defined below.

Simple Weight Function

$$W(p, d_1; \theta) = \begin{cases} 1^* & \text{if } 0 \leq p \leq \theta \\ 0 & \text{otherwise} \end{cases} \quad W(p, d_2; \theta) = \begin{cases} 1^* & \text{if } 1 - \theta \leq p \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Linear Weight Function

$$W(p, d_1; \theta) = \begin{cases} 1^* \cdot \frac{p}{\theta} & \text{if } 0 \leq p \leq \theta \\ 0 & \text{otherwise} \end{cases} \quad W(p, d_2; \theta) = \begin{cases} \left(1^* - \frac{1}{\theta}\right) + \frac{p}{\theta} & \text{if } 1 - \theta \leq p \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

In the definitions of the weight functions, the symbol 1^* denotes one value unit, for example, \$1,000,000. The cost of one trial is denoted by c , where $0 < c < 1^*$. The outcomes of successive trials are considered to be independent.

The methodology for solution of these problems is described in Chapter 4 of Statistical Decision Functions by A. Wald (Wiley, 1950) along with a solution for the case $\theta = 1/4$, $c = .004$, W simple. The solution is presented in the form of a table with upper and lower numerical entries in each cell of the table. Similar tables are presented with these notes for the cases $\theta = 1/3$, $c = .005$, W simple and W linear. Instructions for the use of these tables are as follows. Trials are conducted sequentially and beginning at the extreme upper lefthand cell of the table, the experimenter locates his position in the table by moving one cell to the right for a trial which results in success and one cell downwards for each trial which results in failure; at the first time that a cell is reached where the upper and lower entries in the cell are equal, a terminal decision is reached on the basis of comparison of p_e with θ as described above. The upper entries in the cells of the table represent expected risk (sum of averages of cost of wrong decision and cost of further trials) if a terminal decision is reached without further trials and the lower entries represent the expected risk if trials are continued. Thus, even though the trials should be continued when the upper entry exceeds the lower entry, the experimenter may wish to exercise his judgement regarding further trials at any stage on the basis of the expected risks, rather than follow the optimum procedure. If the experimenter is always required to follow the optimum procedure, he may wish to use simplified tables of the form shown with these notes. With these simplified tables, trials are conducted sequentially until a position marked by X is reached. Either type table permits determination of the minimum and maximum number of trials which will be conducted following the optimum procedure in advance of any testing. Thus an outstanding advantage of this statistical technique is that it provides a quantitative basis for planning and scheduling trials. Figures 17 and 18 presented with these notes show graphs of the maximum number of trials which will be conducted under the optimal procedure as a function of the cost of individual trials, c , for various values of θ for the case of simple and linear weight functions.

a .3333
c .0050

W Simple

.33333	.11111	.03704	.01235	.00412	.00137	.00046	.00015	.00005	.00002	.00001	.00000	.00000
.04397	.03897	.02566	.01235	.00412	.00137	.00046	.00015	.00005	.00002	.00001	.00000	.00000
.11111	.25926	.11111	.04527	.01783	.00686	.00259	.00097	.00036	.00013	.00005	.00002	.00001
.03897	.05060	.04560	.03270	.01783	.00686	.00259	.00097	.00036	.00013	.00005	.00002	.00001
.03704	.11111	.20988	.10014	.04527	.01966	.00828	.00340	.00137	.00054	.00021	.00008	.00003
.02566	.04560	.05245	.04745	.03504	.01966	.00828	.00340	.00137	.00054	.00021	.00008	.00003
.01235	.04527	.10014	.17330	.08794	.04242	.01966	.00882	.00386	.00165	.00069	.00029	.00012
.01235	.03270	.04745	.05233	.04733	.03526	.01966	.00882	.00386	.00165	.00069	.00029	.00012
.00412	.01783	.04527	.08794	.14485	.07656	.03863	.01876	.00882	.00404	.00181	.00079	.00034
.00412	.01783	.03504	.04733	.05116	.04616	.03440	.01876	.00882	.00404	.00181	.00079	.00034
.00137	.00686	.01966	.04242	.07656	.12209	.06645	.03465	.01743	.00850	.00404	.00187	.00085
.00137	.00686	.01966	.03526	.04616	.04929	.04429	.03287	.01743	.00850	.00404	.00187	.00085
.00046	.00259	.00828	.01966	.03863	.06645	.10354	.05762	.03083	.01595	.00801	.00392	.00187
.00046	.00259	.00828	.01966	.03440	.04429	.04678	.04178	.03083	.01595	.00801	.00392	.00187
.00015	.00097	.00340	.00882	.01876	.03465	.05762	.08823	.04996	.02728	.01443	.00742	.00372
.00015	.00097	.00340	.00882	.01876	.03287	.04178	.04358	.03858	.02728	.01443	.00742	.00372
.00005	.00036	.00137	.00386	.00882	.01743	.03083	.04996	.07548	.04335	.02407	.01297	.00681
.00005	.00036	.00137	.00386	.00882	.01743	.03083	.03858	.04065	.03565	.02407	.01297	.00681
.00002	.00013	.00054	.00165	.00404	.00850	.01595	.02728	.04335	.06477	.03764	.02119	.01160
.00002	.00013	.00054	.00165	.00404	.00850	.01595	.02728	.03565	.03796	.03296	.02119	.01160
.00001	.00005	.00021	.00069	.00181	.00404	.00801	.01443	.02407	.03764	.05572	.03270	.01864
.00001	.00005	.00021	.00069	.00181	.00404	.00801	.01443	.02407	.03296	.03541	.03041	.01864
.00000	.00002	.00008	.00029	.00079	.00187	.00392	.00742	.01297	.02119	.03270	.04305	.02844
.00000	.00002	.00008	.00029	.00079	.00187	.00392	.00742	.01297	.02119	.03041	.03280	.02780
.00000	.00001	.00003	.00012	.00034	.00085	.00187	.00372	.00681	.01160	.01864	.02844	.04151
.00000	.00001	.00003	.00012	.00034	.00085	.00187	.00372	.00681	.01160	.01864	.02780	.02975

a .3333
c .0050

W Linear

.16667	.03704	.00926	.00247	.00069
.02466	.01966	.00926	.00247	.00069
.03704	.09259	.02963	.00960	.00314
.01966	.02546	.02046	.00960	.00314
.00926	.02963	.05967	.02254	.00840
.00926	.02046	.02426	.01926	.00840
.00247	.00960	.02254	.04138	.01724
.00247	.00960	.01926	.02206	.01706
.00069	.00314	.00840	.01724	.03000
.00069	.00314	.00840	.01706	.01835

2 .2500
 W simple

c .0001

	SUCCESSES																		
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
F	0					X													
A	1						X												
I	2							X											
L	3							X											
U	4								X										
R	5	X								X									
E	6			.							X								
S	7		X								X								
	8			X	X							X							
	9				X								X						
	10					X								X					
	11						X	X						X					
	12							X								X			
	13								X								X		
	14									X	X						X		
	15											X						X	
	16												X	X					X
	17														X				X
	18															X	X		

c .0010

		SUCCESES											
		0	1	2	3	4	5	6	7	8	9	10	11
F	0					X							
A	1						X						
I	2							X					
L	3							X					
U	4	X							X				
R	5		X							X			
E	6			X	X					X			
S	7					X					X		
	8						X	X				X	
	9								X				X
	10									X			X
	11										X	X	

c .0100

```

      SUCCESSES
      0 1 2 3 4 5 6 7 8 9 10
F      1      X
A      2 X      X
I      3      X      X
L      4      X X
U      5
R      6
E      7
S      8
      9
      10

```

0 .2500
W linear

c .0001

	SUCCESSES											
	0	1	2	3	4	5	6	7	8	9	10	11
F	0				X							
A	1					X						
I	2						X					
L	3							X				
U	4	X						X				
R	5		X						X			
E	6			X						X		
S	7				X	X				X		
	8						X				X	
	9							X	X			X
	10								X			X
	11									X	X	

c .0010

	SUCCESSSES
	0 1 2 3 4 5 6 7 8 9 10
F	0 X
A	1 X
I	2 X
L	3 X
U	4 X X
R	5 X X
E	6 X X
S	7
	8
	9
	10

c .0100

SUCCESSES	
	0 1 2 3 4 5 6 7 8 9 10
F	0 X
A	1 X
I	2 X X
L	3
U	4
R	5
E	6
S	7
	8
	9
	10

0 .3333
W simple

c .0025

SUCCESSSES

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
F	0				X														
A	1					X													
I	2						X												
L	3							X											
U	4	X							X										
R	5		X							X									
E	6			X							X								
S	7				X							X							
	8					X							X						
	9						X	X						X					
	10							X							X				
	11								X							X			
	12									X							X		
	13										X							X	
	14											X							X
	15												X						
	16													X					
	17														X				X
	18															X	X		

0 .3333
W simple

c .0050

		SUCCESSIONS													
		0	1	2	3	4	5	6	7	8	9	10	11	12	13
F A I L U R E S	0				X										
	1					X									
	2						X								
	3	X						X							
	4		X						X						
	5			X						X					
	6				X					X					
	7					X					X				
	8						X	X				X			
	9								X				X		
	10									X				X	
	11										X				X
	12											X			X
	13												X	X	

c .0100

```

      S U C C E S S E S
      0 1 2 3 4 5 6 7 8 9 10
F    0
A    1      X
I    2      X
L    3 X
U    4   X X
R    5      X
E    6      X
S    7      X      X
      8      X      X
      9      X X
     10

```

c .05

	SUCCESSSES
	0 1 2 3 4 5 6 7 8 9 10
F	0 X .
A	1 X
I	2
L	3 .
U	4
R	5
E	6
S	7
	8
	9
	10

a .3333
 W linear

c .0025

SUCCESSSES

[illegible]

c .0050

SUCCESS

[illegible]

c .0100

SUCCESSSES

[illegible]

c .05 .

SUCCESSSES

[illegible]

∂	.4000
W	linear

c .0025

SUCCESSSES

[illegible]

c .0050

SUCCESSSES

[illegible]

c .0100

SUCCESSSES

[illegible]

c .0500

SUCCESSSES

[illegible]

Figure 18
 Plot of Maximum Number of Trials, N_M ,
 against
 Cost of Testing, c ,
 for indicated values of δ .
 (Simple Weight Function)

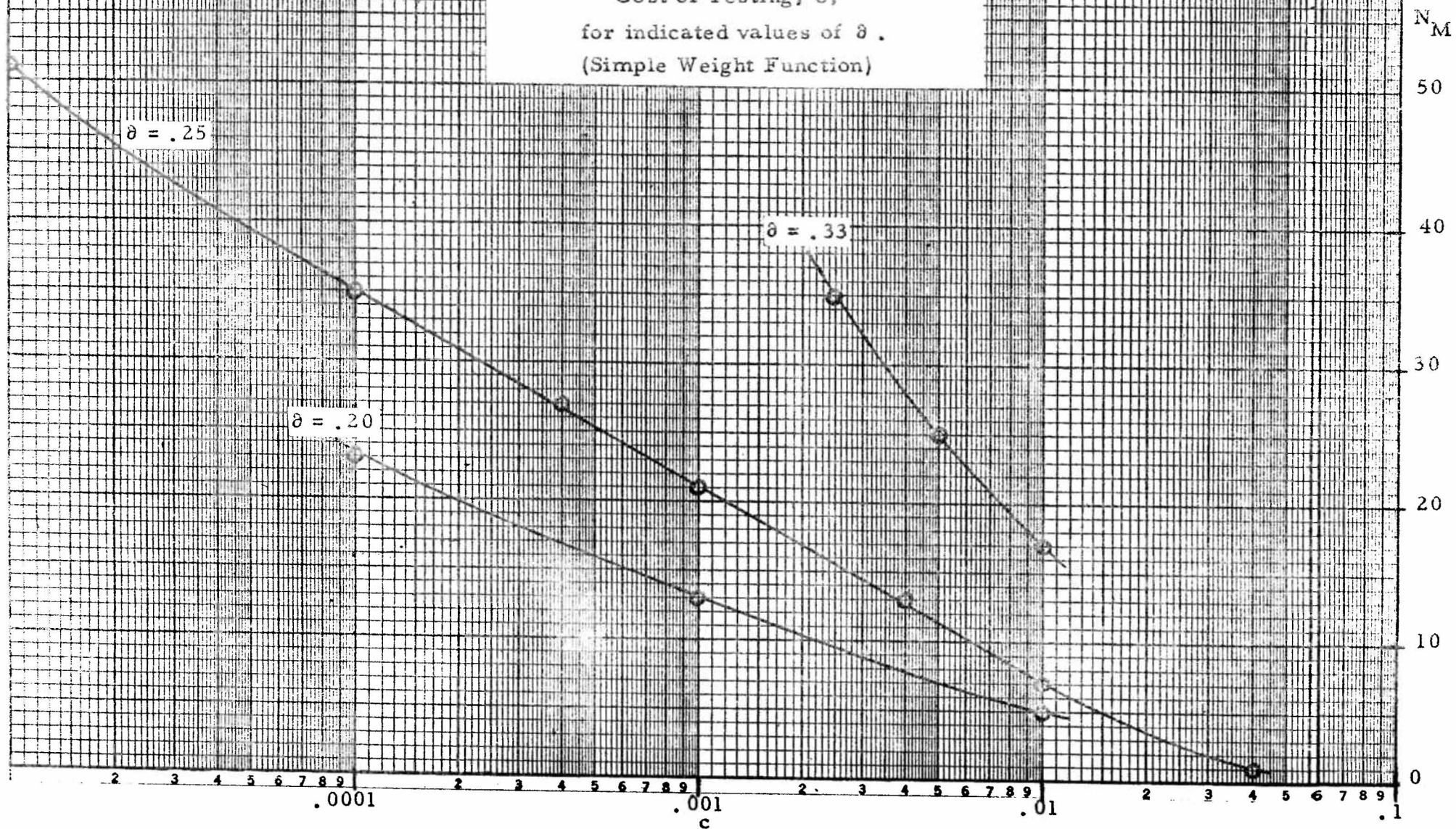


Figure 17
 Plot of Maximum Number of Trials, N_M ,
 against
 Cost of Testing, c ,
 for indicated values of θ .
 (Linear Weight Function)

